**Assessment of stability of network logistics business**

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**Abstract**

Digital logistics platforms are based on technologies for inter-machine exchange of information flows online and decision-making algorithms for processing the incoming data of an economic nature. While there is a full range of hardware available for equipping all links of logistics managemetn, including servers, peripheral sensors, adapters for high-speed mobile networks, and cloud platforms, algorithmic (intelligent) software, or Brainware, still remains the subject of discussion. This study considers the stability of the behavior of mathematical models characterizing the transfer processes in networks with complex structure. This problem is highly important for logistics of both a wide range of discrete cargo, and for quasi-continuous cargo, for example, liquid hydrocarbons. An integral functional is taken as the criterion of the logistics operator. The solution is obtained based on a system in the class of integrable functions that adequately describe the transfer of masses with a complex internal rheology. A generalized solution of the system is a function that defines the variational statement of the initial boundary-value problem. The consideration is extended beyond the framework of classical continuously differentiable solutions because the physical nature of transfer of goods and cargo flows through logistics networks has to be described more accurately.

**Keywords**: Logistics, digitalization, mathematical model, sustainability, optimization

**1. Introduction**

Building a sustainable supply chain means balancing reliability and flexibility. Both of these properties of the supply chain can act as a safety cushion against uncertainties and should be taken into consideration when planning. Legacy logistics solutions, have not justified themselves in the digital age. Decentralized communication is giving way to more precise and modern solutions. The combination of optimization modeling and data analysis is a basic technology that allows you to create a fairly detailed model of the real supply chain of its digital counterpart Idea. The goal of creating a digital twin is to manage the risks in the supply chains, thereby making them more reliable and resilient in the event of any failures. Modern technologies allow you to collect large amounts of data on supply chains online. The management technologies used in supply chains enable the identification of critical hot spots and timely warning of incidents that may have a critical impact on the supply chain (Barykin, Bochkarev, Kalinina and Yadykin 2020). It can successfully act as a replacement for expensive field monitoring methods, without causing any damage to the physical object. A digital twin refers to an evolving digital profile of the historical and current behavior of a physical object or process that helps optimize business performance (Kapustina et al. 2020).

Rapid changes in the field of high technology leads to changes in all spheres of life. Enterprises need to transform their workforce to achieve a sustainable competitive advantage. The introduction of high-tech solutions leads to a change in the format and content of almost any work, as well as a rethinking of approaches to personnel planning (Bril, Kalinina et al. 2020). When developing a business plan for a new enterprise, the main issues are market assessment and-sales forecast opportunities to attract investment. When drawing up a business plan for an existing enterprise, another important issue arises - the use of internal resources of the enterprise, assets and personnel. Therefore, when developing an enterprise plan, there is a need for a special financial and economic analysis of the object and the preparation of key financial indicators of business planning standards (Bril, Evseeva et al. 2020).

It is necessary to consider the issue of training personnel for the digital industry in the context of creating teams at enterprises in scientific organizations and universities that have the necessary digital competencies and digital literacy, that is, the ability to use modern methods, experimentation and research technologies to develop globally competitive products. This competence becomes an integrator of the technological, ethical, and cognitive aspects of education. In this paradigm, the basis of the methodological approach to solving the problem of personnel training is a problem-oriented scientific approach, and the model of creating mirror engineering centers is a possible model for scaling competencies, spreading knowledge and developing network interaction (Barykin, Borovkov, Rozhdestvenskiy, Tarshin et al. 2020).

A digital ecosystem is a self-organizing, sustainable system with digital platforms at the base, which form a single information environment where the members of the ecosystem can interact when no hard functional ties between them exist. The base of a digital ecosystem is the diverse platform technologies which form a single information environment for all members: society, business and government. The development of platform technologies contributes to the scaling up of digital ecosystems, which are beyond the business environment and B2B and B2C relationships. The digital environment is used to connect members, exchange information and resources, organize processes and coordinate objectives. One of the most important features of a DES is its ability to self-organize and the lack of a managing or supervising member of the ecosystem. The main difference between digital ecosystems and traditional ecosystems is that the organization of business in the latter is based on management decision making by a human. Meanwhile, management decision making in digital ecosystems is for the most part automated and carried out without human participation, thanks to the diverse tools which exist such as artificial intelligence, computer vision and so on (Barykin, Kapustina, Kirillova, Yadykin et al. 2020).

The process of managing the organization's personnel within the digital ecosystem, while maintaining the basic goal setting, is transformed from an instrumental point of view. The transition from a traditional HR system to a digital one involves a difference between how staff interact with each other and with management. The key factor in the digital interaction system is the ability of each employee to freely contact the information and analytical center. The digital interaction system also differs from the traditional one by expanding the channels for exchanging information and analytics. Considering the tasks of personnel management in the digital era, it can be shown that the problem of digital personnel management is the scientific search for the features of employee interaction in the new technological structure of Industry 4.0 under the influence of the flow of innovations, there is a change. It includes improvements in performance, reliability, quality, and safety (Barykin, Kalinina, Aleksandrov, Konnikov et al. 2020).

Over the past decade, technologies and equipment that can radically change the approach to managing the processes of cargo movement and cargo transportation have become widespread. This includes the possibility of machine-readable exchange of information about goods and cargo among themselves. The structure of relations between participants in this segment of the economy has become more complex. This is due to the processes of consolidation in commercial structures, which leads to an increase in the complexity of managing such business objects, as well as a multiple increase in the time and money spent on ensuring the material flows of goods and cargo necessary to maintain a competitive business. To solve such large-scale problems, the use of scientifically based mathematical models is required. This approach will allow us to use the digital footprint of processes in the logistics network and develop an algorithmic basis for a whole range of software tools for more efficient management of the logistics industry (Barykin, Kapustina, Sergeev and Yadykin 2020).

It is necessary to find an economically optimal management of the flows of goods and cargo based on digital information about their movement, taking into account the stochastic nature of the volumes and directions of flows. At the same time, it is necessary to choose and justify the methodology for calculating the node DC terminals included in the network structure of logistics activities. The obtained set of mathematical relations allows us to simulate the operation of a network distribution center as a node element of a logistics system that functions on the principles of 3PL operator in conditions of stochastic nature of incoming and shipped goods flows (Shmatko et al. 2021).

Mathematical modeling of evolutionary processes of transfer of different goods and cargoes, both discrete and quasi-continuous, is considered in a number of fundamental studies (Zhabko, Shindyapin and Provotorov 2019). One of the main requirements imposed in real logistics business is that the stability of the behavior of mathematical models is to be assessed for finding the potential applications of the obtained results (Borisoglebskaya et al. 2019). There are multiple reasons for this, related to both the real situation and the instability of external conditions. In particular, the input data initiating the process are given with a certain error; in addition, indicators that are important in logistics, such as traffic, the throughput of supply chain segments, waiting times for loading and unloading, customs procedures for cross-border transportation, and other factors, can also be estimated only by random distribution functions (Zhabko, Nurtazina, and Provotorov 2019).

The standard approaches relied on classical methods for studying the solution stability in systems of differential equations, introduced by Lyapunov. The solution was assumed to be stable if it changed little for small perturbations of the initial condition and for any moment of time.

Several studies, for example, (Provotorov 2008) proposed mathematical models of logistic processes in networks using methods combining graph theory and equations of mathematical physics, including the well-developed tools of the theory of optimal decision-making (Podvalny, Podvalny and Provotorov 2017). In this paper, we provide the stability conditions for the solution of an evolutionary parabolic system with distributed parameters on a graph which is the prototype of the logistic transport network (Provotorov and Provotorova 2017). At the same time, the mathematical model reflects the evolutionary process of the transfer of goods and cargo along a spatial network (Artemov et al. 2019). The parabolic system is considered in the class of integrable functions that adequately describe the transfer of masses with a complex internal rheology. A generalized solution or weak solution of the system in this case is an integrable function that determines the variational statement of the initial boundary-value problem.

Accordingly, the answer to the question about the influence of changes in the initial data on the behavior, and analysis of the mathematical model (primarily the solutions of the initial boundary-value problems governing the model) takes on an additional importance from the economic standpoint. Thus, in general, the issue at hand is risk assessment of investing funds and the stability of the business model.

**2. Problem statement**

The main results in the mathematical theory of stability are well developed in the class of ordinary differential equations. However, because there is a requirement to adequately describe economic processes where the efficiency criterion contains at least two variables, one of which is time, these differential systems have to be abandoned in numerous applications in favor of considering evolutionary partial differential equations, due to the complexity of mathematical models. This practically important case is analyzed in our study. We consider the stability of an evolutionary system with distributed parameters, generating a mathematical model, on a graph (network) with unlimited increase in the time variable.

**3. Main formalisms**

We use the definitions and notations adopted in (Provotorov, Sergeev and Part 2019). The logistics network is illustrated with a graph of arbitrary rheology, which is a bounded directed geometric graph with the edges , parameterized by the segment. Here  and  denote the sets of boundary  and internal  nodes of the graph;  is the union of all edges of  that do not contain endpoints;  (),  (,  is an arbitrary fixed constant).

The Lebesgue integral over oris used throghout the study:

 or ,

 corresponds to the restriction of the function to the edge.

Let us introduce the definition of the required spaces and sets:  is the space of continuous functions on is the Banach space of functions measurable on, summable with *p*th degree; spacesare defined similarly;

is the space of functions fromwith the norm defined by the ratio



We use the equivalents of the Sobolev spaces (Volkova, Gnilitskaya and Provotorov 2014): is the space of functions from with a generalized first-order derivative also belonging to (Provotorov, Ryazhskikh and Gnilitskaya 2017); is the space of functions from with a generalized first-order derivative with respect to belonging to (the space is introduced similarly); is the set of all functions  with a finite norm:

 (1)

and continuous with respect to in the norm , i.e., such that given ,  is uniform on.

Let us introduce the space of states of a parabolic system, and auxiliary spaces. For this purpose, let us consider a bilinear form:

 (2)

with fixed measureable functions , which are bounded on , summable with a square:



**4. General mathematical model**

Using the terms introduced for modeling the movement of goods along the logistics network structure, we can adopt a rigorous mathematical formulation to translate the processes to a form accessible for algorithmic description. We apply the following statement (Provotorov and Provotorova 2017). Let the function  be such that  for any  (is a fixed function). Then, for any edge, the restriction  is continuous at the endpoints of the edge . Let us denote by  the set of functions  satisfying these conditions and relations:



at all nodes of the graph, which is the image of the logistics network  (here  andare the sets of edges γ, oriented, respectively, <<towards the node >> and <<away from the node>>). The closure of the setin the norm  is denoted by. If we assume that the functions from also satisfy the boundary condition , then we obtain the space . Next, let be the set of functions , whose traces are defined on the sections of the region  by the plane as functions of the class  and satisfy the relations:

 (3)

for all nodes of the logistics network . The closure of the setin the norm (1) is denoted by; from here it obviously follows that the condition  is satisfied.

Another subspace of the space  is , closing the set of smooth functions satisfying the relations (3) for all nodesand for anyin the norm  (the space is introduced similarly); the absorption condition  is satisfied.

Thus, the space  describes the set of states of a parabolic system, andand describe auxiliary spaces within the framework of this approach. The difference between the elements of the space and the elements of  is that the latter have no continuity with respect to the time variable *t*. This is fundamentally important for generating economic indicators characterizing the transfer of goods. Indeed, an integrated criterion reflecting the costs of transferring goods depends both on the distance traveled and on time. Such dependences exhibit jumps due to changes in tariffs for multimodal transportation, approaching the expiry date of a wide range of fast-moving consumer goods (FMCG), transport lease conditions, port infrastructure, demurrage, detention, or contractual penalties for short delivery or late shipment, and other factors accompanying logistics operations.

Let us consider the following parabolic equation in the spaces  and :

 (4)

representing a system of differential equations with distributed parameters of the economic criterion on each edge  of the graph  as an image of a logistics network;. The state  of system (4) in the region  is determined by a generalized solution  of Eq. (4), satisfying the initial and boundary conditions:

 (5)

. Above, we have already established that the functions and are summable with a square. It follows from the condition that the mapping of the economic criterion to the graph is a continuous function, so that the first equality in (5) holds true and is definable almost always. In this case, we consider the first initial boundary-value problem (4), (5) or the Dirichlet boundary condition in relations (5).

Let us present the main statements and steps for the proof of the validity of the results obtained, necessary for the study of solution stability. The main practical applications are analysis of management strategies adopted by managers of logistics services and risk management of logistics business processes. Let us first consider the solution in the auxiliary space, then in the space . Complete proofs are given in (Volkova and Provotorov 2014).

Let us define that a generalized solution of the initial boundary-value problem (4), (5) of class  is a function that satisfies the integral identity:

 (6)

for any equal to zero at; is the bilinear form defined by the relation:

;

Determining the solvability of problems and (4), (5) in the space  (and ), a special basis of the space  is used, which is a system of generalized eigenfunctions of the boundary-value problem for eigenvalues (spectral problem):

 (7)

in the functional class (Golosnoy et al. 2019).

It follows then that, assuming (2), the spectral problem (7) has a countable set of real eigenvalues (numbered in ascending order with respect to their multiplicities) with a limit point at infinity (the eigenvaluesare positive, except possibly a finite number of the first ones). The system of generalized eigenfunctions  forms a basis in  and , orthonormal in and orthogonal in the sense of the scalar product (Provotorov et al. 2020).

Since we take into account the economic meaning of variables in practical applications, it follows from the condition that  should be non-negative that all eigenvalues of spectral problem (7) are positive. The positivity of the eigenvalues is the determining factor for establishing the stability property of parabolic evolutionary systems with distributed parameters on the graph.

**5. Study of solution stability**

Let us carry out a mathematically substantiated study for the stability parameters of the solution found for system (4), which describes the dynamics of the logistic process, formalized as the transfer of goods and cargo over the network. The transport network, in turn, is presented in the form of a mathematical graph. Suppose that  for , which guarantees that the eigenvaluesare positive. Let us consider the resulting system (4) on the set . Let us introduce the notations,  (), , ; evidently, . As above, , with  for any .

Let the state of system (4) be described by the function , which is a generalized solution of Eq. (4) in the region  with the initial and boundary conditions:



the function  is a weak solution of Eq. (4) in the region  with the initial and boundary conditions:



The state  of system (4) is assumed to be unperturbed, and that of is assumed to be perturbed. It follows from previous analysis of the generalized solution of problem (4), (5)) that the states , are defined in the region , satisfy the corresponding initial and boundary conditions, and belong to the space  at.

Below, we propose the definition for the stability of a solution to the initial boundary-value problem (4), (5) (stability of the unperturbed state of system (4)) as an equivalent of Lyapunov stability.

Let us assume that the unperturbed state  of system (4) is called stable if for any and , there is , such that at  is fulfilled at , where is the perturbed state of system (4).

Similarly to the definition for the stability of the unperturbed state of system (4), we can introduce the definition for the uniform stability of the unperturbed state of system (4) in the region . For system (4), we can introduce definitions of the asymptotic and exponential stability of the unperturbed state of system (3) in the region . Because system (4) is linear, all definitions can be reformulated for the zero (trivial) state of the system (4).

Let us prove that given the above condition , the unperturbed state of the system (4) is stable in the region. This follows from the fact that, due to the linearity of equation (4), the functionis an element of the space  and is a generalized solution of the initial boundary-value problem for the homogeneous equation (4), satisfying the initial and boundary conditions:



where . Therefore, the initial boundary-value problem (4) is uniquely generalized solvable, its solution has the representation,  and is the limit of the weakly converging sequence of  approximations:



and the following inequality holds simultaneously:

.

Considering the limit in the last inequality at, we obtain the estimate:



for any;  is a constant independent of . The latter means that



Let us fix , taking, then, since



the inequality  follows for any , which was required to obtain the stability conditions.

**6. Conclusion**

Data flow processing algorithms reflecting the current state of cargo movements are the core of digital logistics platforms. The efficiency of a logistics operator directly depends on the quality of management decisions made. Since the operation of all links of a logistics chain is influenced by random perturbations, it is necessary to study the stability of the obtained solutions to confirm the practical possibility of switching to leading planning indicators. The technique proposed in the paper is a continuation of the standard approaches using classical methods for studying the solution stability in systems of differential equations. However, the dynamics of network processes of goods transfer plotted on graphs with different topologies requires mathematical models associated with solving initial boundary-value problems in the class of differential equations with respect to the distributed parameters. This reflects the conditions imposed on the solution of equations describing systems with parameters distributed on the graph. In this paper, we provide the stability conditions for the solution of an evolutionary parabolic system with distributed parameters on a graph which is the prototype of the logistic transport network. We used a quality functional integrating the economic indicators of the process depending on time and on the parameter of goods transfer. Solving the problem posed and applying a generalized solution is a complicated task because the quality functional is not continuous with respect to its arguments. This is due to the real conditions of logistics operations. The most typical ones are inevitable time delays and stevedoring costs for loading and unloading operations, customs procedures, changes in mileage rates for multimodal transportation, the influence of the time factor on the costs of renting transportation services and port/station infrastructure, demurrage, detention, or contractual penalties for short delivery or late shipment, stringent restrictions on the delivery schedule for most FMCG goods and other types of goods with short expiry dates. Furthermore, other perturbations can have a considerable effect on economic indicators, such as congestion of transport routes and variable density of average traffic, sensitivity to adverse weather conditions, especially in the air and sea transportation segment, as well as possible complications of a political nature, for example, sanctions and force majeures, gaining a particularly prominent role during the COVID-19 pandemic. All these are components of problems in supply chain management, increasing in complexity in the context of business consolidation into network structures. The results obtained allow to estimate the influence of changes in the initial data on the behavior and analysis of the mathematical model, primarily the solutions of the initial boundary-value problems that determine the model. This theoretical solution gains a much more important economic meaning. It can be concluded that, in general, the issues to be considered are risk assessment of investing funds and the sustainability of the business model used by the logistics operator.

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